



ELEN E3106/4106 Lecture 4

Charge Carriers in Semiconductors and Intro to Fermi Level Outline

- Electrons and holes
- E - k diagrams
- Effective mass
- Intrinsic and extrinsic materials
- Thermal equilibrium
- Fermi level

Assignments:

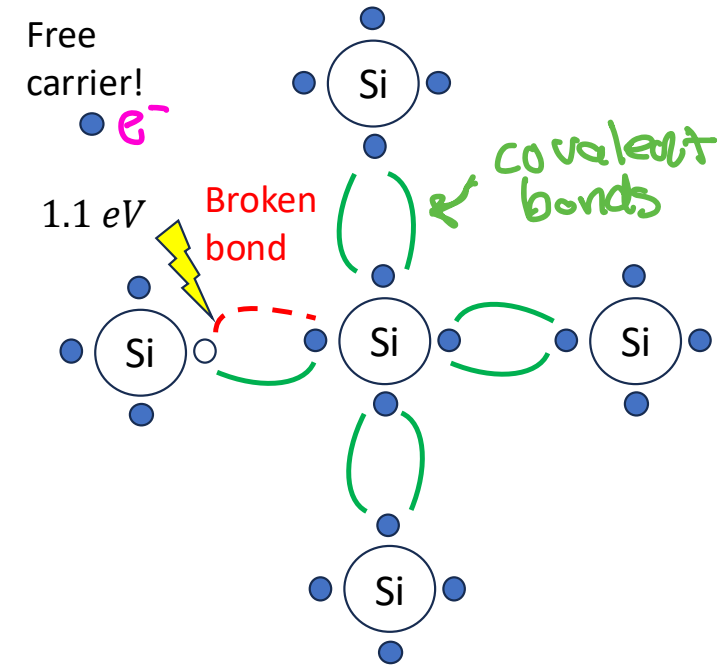
Reading: Streetman and Banerjee §3.2, 3.3.1

Homework 1 due TOMORROW Friday Sept 12th by 5pm

Homework 2 due Friday Sept 19th by 5pm

Bond Breakage and Free Carrier Generation

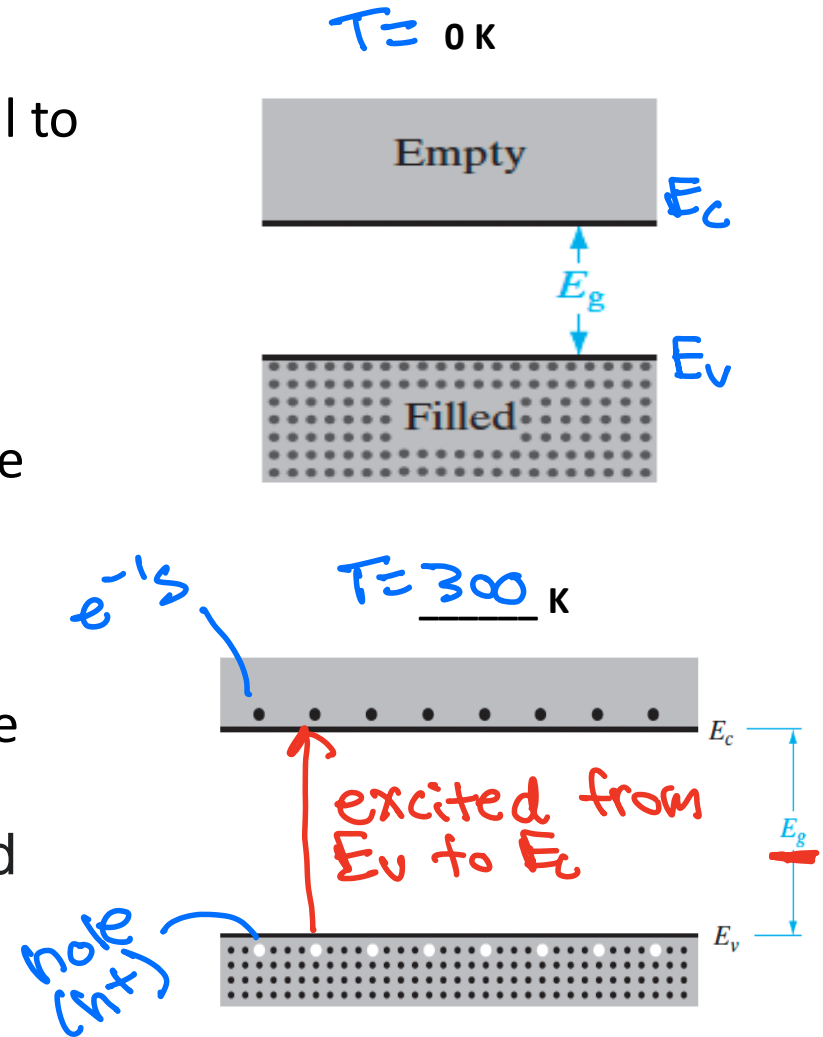
- Breaking the bonds in a crystal requires a certain amount of energy
- This energy is **equal to the** E_g , bandgap
- If you invest this energy into the lattice, you can kick an e^- out of an otherwise satisfied bond state
- Now, the e^- will rattle around in the crystal as a free carrier
- This process is called free carrier generation



For a well written paragraph on the broken bond model of EHP generation, check out Bhattacharya, *Solid State Electronic Devices* pg. 76 (see syllabus)

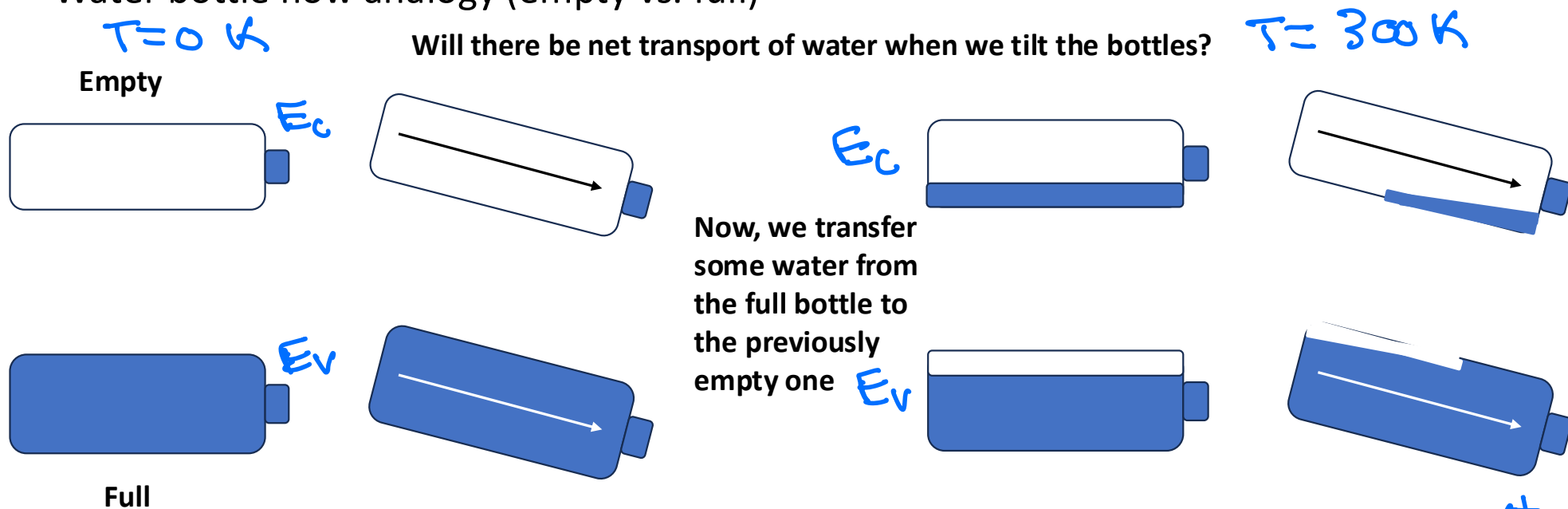
Carriers: Electrons and Holes

- Above 0 K, e^- has absorbed enough energy (equal to E_g) to break free of its bond
- This means it has enough energy to leave E_v and “jump” into E_c
- The ‘free’ e^- will leave behind a hole
- Holes are the empty electron states in the valence band and behave as particle
- Hence, electrons and holes are generated as electron-hole pair (EHPs)
- e^- carry negative charge ($-q$) and h^+ carry positive charge ($+q$)
- h^+ can move through the lattice as e^- can, and act similarly to positively-charged particles.



Electrons, Holes, and Net Current

- There can be no net current in a filled band unless an electron is removed
- In other words: *For e^- to be accelerated in an electric field, they must be able to move into new, unoccupied states*
- Water bottle flow analogy (empty vs. full)



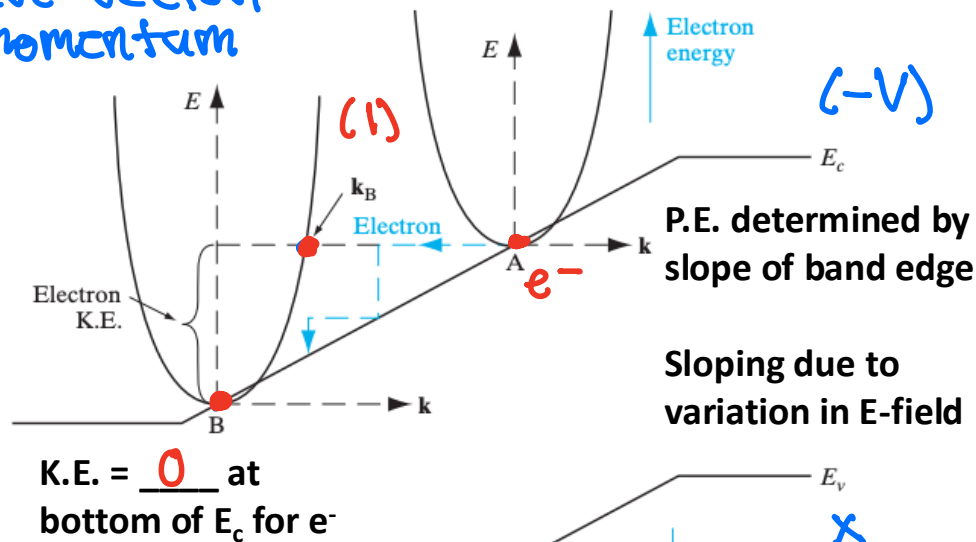
Takeaway: An empty E_c devoid of e^- of a completely full E_v cannot give rise to a net movement of e^- , so there is no current conduction

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*, E. Pop's ECE 340 lecture

E-k and Energy Band Diagrams

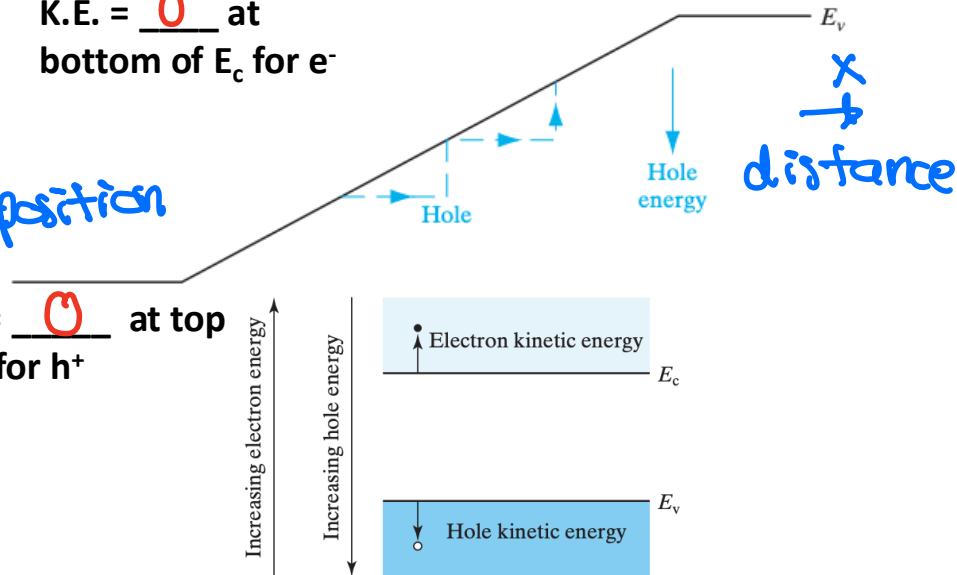
k is wave vector,
crystal momentum

- Let's combine energy bands vs. k and vs. x
- A band diagram plots the energy levels of e^- within a material as a function of distance
- The slope of the band edge at a given point tells you how the potential energy changes with distance
- Electric field relationship: E-field is the negative gradient of PE, a (+) slope on the band diagram indicates a (-) electric field
- By analyzing the slope of a band diagram, engineers can understand how free carriers will move through a semiconductor device!



- Note which way energy of holes increases
- Both e^- and h^+ tend to seek their lowest energy position
- e^- tend to fall in the energy band diagram
- h^+ float like bubbles of water
- 1) e^- at A gains KE and loses PE, moves to nonzero wavevector k_B
- 2) e^- loses KE to heat through scattering, returns to B, bottom of band

K.E. = 0 at top of E_v for h^+



The Concept of Effective Mass

- What is mass? Describes how a particle resists changes in motion
- Recall: The electrons and holes in a crystal interact with a periodic Coulombic field in the crystal (Bloch's theorem)
- They surf over the periodic potential of the crystal, and therefore m_n and m_p are not the same as the mass would be in free space
- Denoted m_n^* for e^- and m_p^* for h^+ . 'n' for negative charge carrier', 'p' for positive charge carrier
- We have different effective masses for different materials and purposes

Conductivity
effective
mass

TABLE 1-3 • Electron and hole effective masses, m_n and m_p , normalized to the free electron mass.

	Si	Ge	GaAs	InAs	AlAs
m_n/m_0	0.26	0.12	0.068	0.023	2.0
m_p/m_0	0.39	0.30	0.50	0.30	0.3

$$m_0 = 9.11 \times 10^{-31} \text{ kg}$$

Note: Recall periodic potential is usually anisotropic, so m^* depends on direction. But, we can use appropriate average (shown here) in most calculations.

Sources: C. Hu, *Modern Semiconductor Devices for Integrated Circuits*

Intrinsic Material

- If there are no dopants present, the semiconductor is called intrinsic
- Recall: No free charge carriers at 0K --> E_c empty, E_v filled
- Recall: Above 0 K, EHPs are thermally generated and we have free carrier generation

- We can quantify the # of e^- and h^+ in the semiconductor
 - Temperature dependent!

e^- / volume

n (cm^{-3}) electron concentration

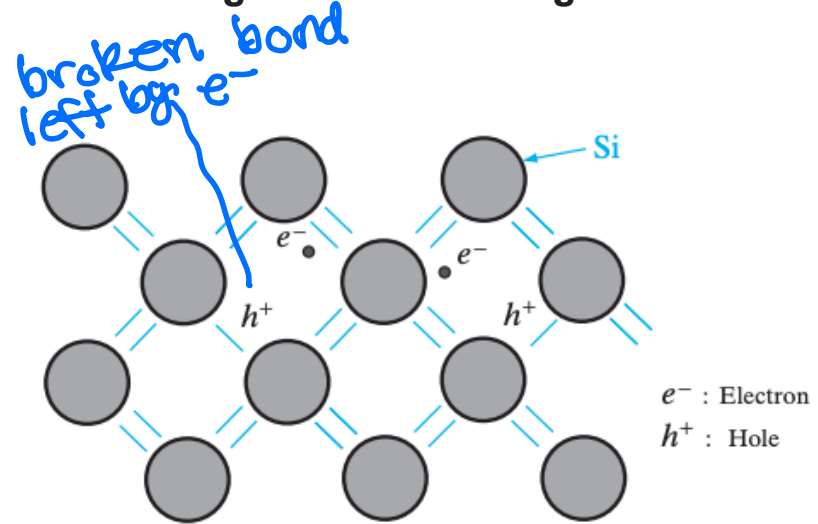
p (cm^{-3}), hole concentration

n_i (cm^{-3}), intrinsic carrier concentration of EHPs -> this is a temp. dependent **material property**

$$n = p = n_i$$

$$np = n_i^2$$

Important: When you generate a free carrier through EHP generation, you do not generate net charge!



1:1 ratio in free carrier generation


At room temperature ($T \sim 300$ K) intrinsic concentrations: Ex:

- $n_i \sim 2 \times 10^6$ electrons and holes per cm^3 in GaAs
- $n_i \sim 1.5 \times 10^{10} \text{ cm}^{-3}$ in Si
- $n_i \sim 2 \times 10^{13}$ in Ge

GaAs: 1.4 eV
Si: 1.1 eV
Ge: 0.7 eV

What about the band gaps of these materials?

To get a sense of scale, how do these numbers:

- compare with N_A ?  # of dopants, $6 \times 10^{20} \text{ atoms/cm}^3$
- compare with number of Si atoms per cm^3 (x4 valence electrons per atom)?
 $5 \times 10^{22} \text{ atoms/cm}^3 \times 4 \text{ valence } e^- = 2 \times 10^{23} e^-/\text{cm}^3$

What if temperature is increased?

$$T \uparrow \quad n_i(T) \uparrow$$

Generation and Recombination

- **Steady-state:** If a semi has been a certain way for a 'long' time (none of the properties are changing with time)
- At steady-state, the rate of recombination of EHPs must be the same rate as they are generated
 - Function of temperature

$$r_i(T) = g_i(T)$$

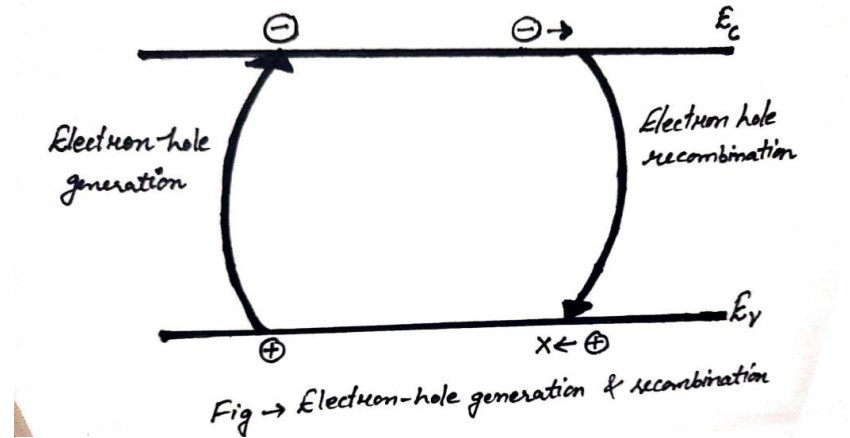
n_0 (cm^{-3}), equilibrium electron concentration

p_0 (cm^{-3}), equilibrium hole concentration

α_r (cm^{-3}), proportionality constant

$$r_i = \alpha_r n_0 p_0 = \alpha_r n_i^2 = g_i$$

As temperature increases, so does g_i and therefore r_i



Extrinsic Material and Doping

- So, intrinsic carriers are created through thermal generation
- How can we manipulate the carrier concentrations?
- We can create carriers in a semiconductor by intentionally adding impurities, called dopants, in a process called **doping**
- The semiconductor is now extrinsic. There are two types:
 - n-type -> excess of e^- -> impurities are called donor because they donate an e^-
 - p-type -> excess of h^+ -> impurities are called acceptors because they accept an e^-
- Equilibrium carrier concentrations no longer equal the intrinsic carrier concentration

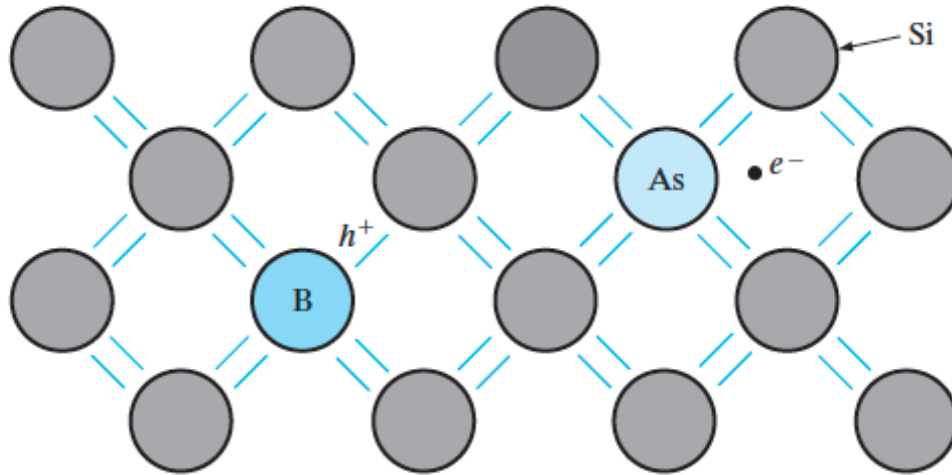
$$n_0 \neq p_0 \neq n_i$$

n-type: maj of e^-
min. of h^+

- Carrier type in excess called the majority carrier
- Other carrier type called the minority carrier

Visualizing donor and acceptor atoms

- Group III and V are common dopants in Si. Why?
 - Group V atoms have 4 valence e^- to satisfy covalent bonds with neighboring Si, plus 1 extra e^- that doesn't fit and is loosely bound (e.g. easily given up)
 - Group III has 3 valence e^- , introducing a hole where the 4th e^- should be



Groups

IIIA	IVA	VA
Boron 5 B 10.811	Carbon 6 C 12.011	Nitrogen 7 N 14.007
Aluminum 13 Al 26.982	Silicon 14 Si 28.086	Phosphorus 15 P 30.974
Gallium 31 Ga 69.723	Germanium 32 Ge 72.64	Arsenic 33 As 74.922
Indium 49 In 114.82	Tin 50 Sn 118.71	Antimony 51 Sb 121.76

P-type Dopants

N-type Dopants

Elemental Semiconductors

Ionization Energy

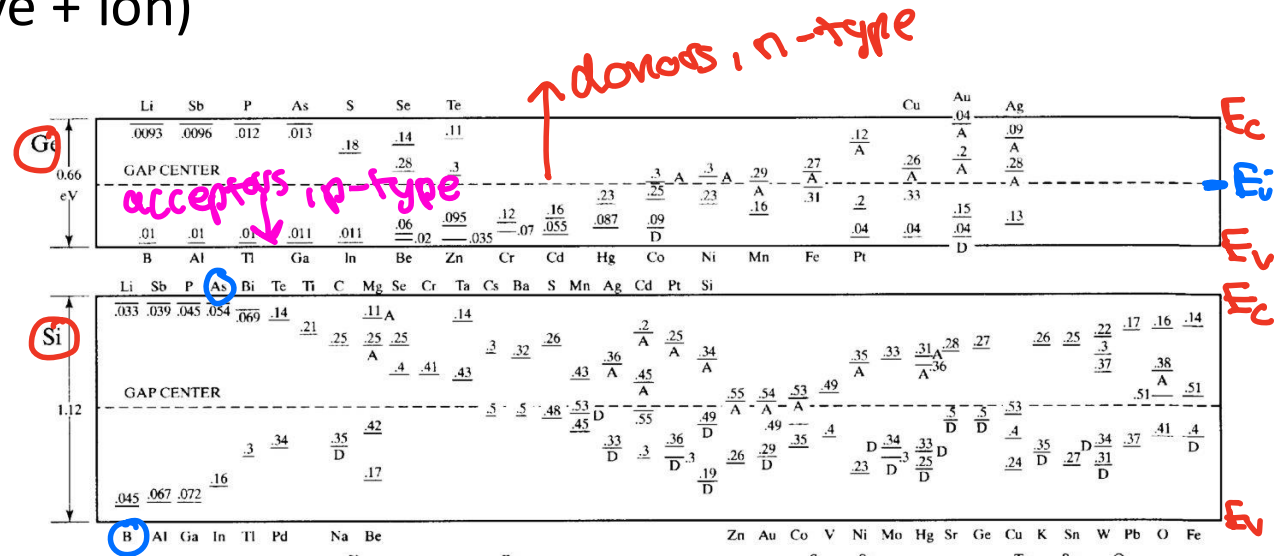
- We can calculate the approximate energy required to excite the 5th e⁻ of a donor atom into E_c (or, for acceptor to accept an e⁻)
- Called the **ionization or binding energy** --> E needed to ionize a donor atom (i.e. free the extra e⁻ and leave + ion)

$$E_{ionization} = \frac{m_n^* q^4}{2(4\pi\epsilon_r\epsilon_0)^2 \hbar^2}$$

ϵ_r - relative dielectric constant of the semiconductor

ϵ_0 - vacuum permittivity (8.85×10^{-12} F/m)

\hbar - reduced Planck constant, $\frac{h}{2\pi}$, (1.05×10^{-34} J·s)



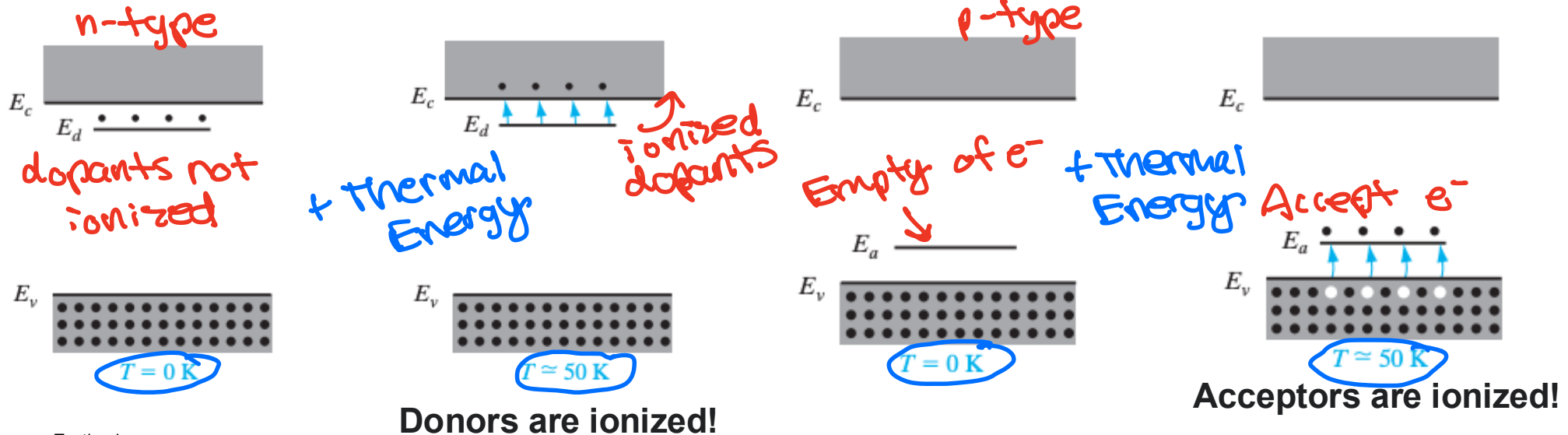
Ionization energies of dopants in Si and Ge. Center is intrinsic Fermi level

Dopant Impurity Levels

At room temperature (300 K):

- n-type $\rightarrow N_D \gg (n_i, p_0)$
- p-type $\rightarrow N_A \gg (n_i, n_0)$
- Dopants introduce additional levels into the energy band structure, usually within the band gap
 - These impurity levels are called E_d for donors, and E_a for acceptors
 - Dopants can be easily 'activated' with a small amount of thermal energy

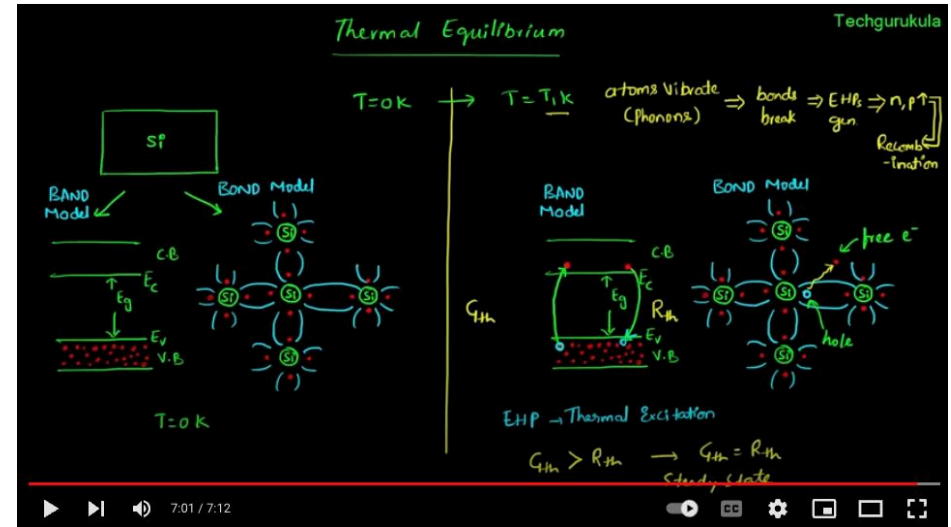
N_D - concentration of donor atoms
 N_A - concentration of acceptor atoms



Thermal Equilibrium

- Equilibrium is the lowest energy configuration in the presence of thermal agitation
 - E.g., the steady-state condition at a given temperature without any external excitation, such as light, E-field, pressure
- EHP generation and recombination is **only** due to thermal excitation
- I recommend this video here for further explanation:

<https://www.youtube.com/watch?v=C0EK5OIXPoQ>



Summary (So Far)

- Band gap energy (E_g) is energy required to free an electron from a covalent bond
 - $E_g = 1.1$ eV for Si at 300 K
 - Insulators have “large” E_g , semiconductors have “smaller” E_g
 - EHPs are generated when bond is broken
- There must be available states for e^- to move into for current flow!
- e^- and h^+ have some effective mass in a given material, dependent on crystal periodicity
- Intrinsic materials are pure, with an intrinsic carrier concentration: $np = n_i^2$
- Extrinsic materials: those with dopants. For Si:
 - Substitute pre-existing Si atoms on lattice sites
 - Group-V elements are donors, contribute conduction electrons
 - Group-III elements are acceptors, contribute holes
 - Low ionization energy (~ 50 meV) \rightarrow all ionized at room T
 - Useful dopant concentrations in Si range from 10^{15} to 10^{20} cm^{-3}

Source: Textbook

Our Goal

- Be able to calculate electron (n) and hole (p) densities for:
 - Any temperature!
 - Any doping concentration!
 - Any energy level!

- Recall: In thermal equilibrium, $np = ni^2$
 - EHPs are generated
 - Recombination rate is equal to generation rate

$$n + \Delta n; p + \Delta p; \Delta n = \Delta p$$

- We are (typically) dealing with large concentrations, not individual electrons \rightarrow we need a statistical treatment of these electron (or hole) populations
- Two key concepts needed to “count” populations:
 - 1) The probability of finding electrons (or holes) in a state
 - 2) The number (i.e. density) of states available

1) Fermi-Dirac probability function for electrons & holes

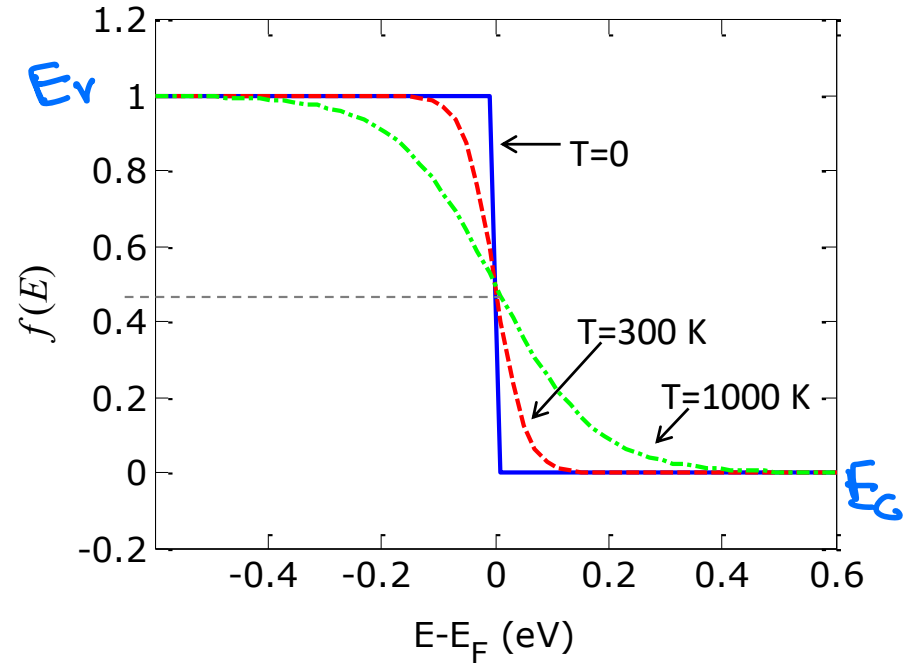
Analogy:

≈ the probability of finding a spectator in a seat as you go up from the bottom row of a stadium

$$0 \leq f(E) \leq 1$$

$$f(E) = \frac{1}{1 + e^{(E - E_F) / kT}}$$

$[1 - f(E)]$ = probability of finding h^+ in given state



Fermi-Dirac Distribution Function

- Electrons in solids obey Fermi-Dirac statistic
- There is a distribution of e^- over a range of allowed energy levels at thermal equilibrium, given by:

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

- The energy state with $\frac{1}{2}$ probability of being occupied by an e^- is called the Fermi level, E_F
- $[1 - f(E)]$ is the corresponding probability that a state is *not* being occupied by e^- (i.e. being occupied by a hole)

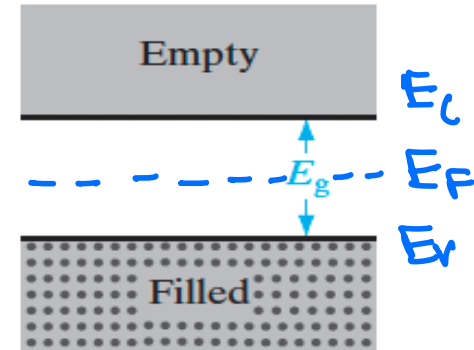
$$0 \leq f(E) \leq 1$$

$$\underline{T = 0 \text{ K:}}$$

$$1) f(E > E_F) = \frac{1}{1 + e^{\frac{+ve}{0}}} = \frac{1}{1 + e^{+\infty}} = 0 \quad E_C$$

$$2) f(E < E_F) = \frac{1}{1 + e^{\frac{-ve}{0}}} = \frac{1}{1 + e^{-\infty}} = 1 \quad E_V$$

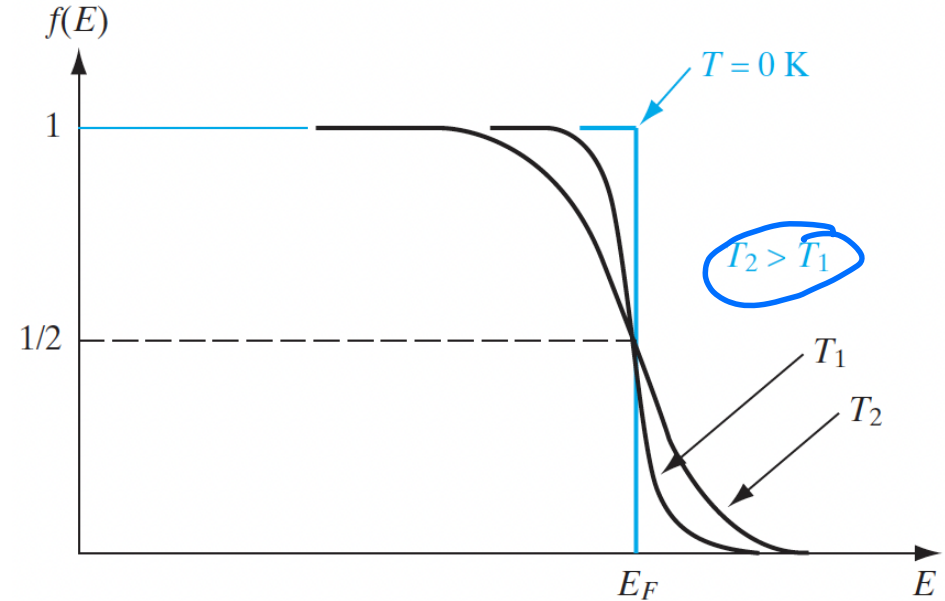
$$3) f(E = E_F) = \frac{1}{1 + e^{\frac{0}{kT}}} = \frac{1}{1 + 1} = \frac{1}{2} \quad E_F$$



The Fermi Level

- E_F is in the bandgap between E_c and E_v
- Important: $f(E)$ is the probability of occupancy of an available state at E
- If there is no **available** state at E (e.g., in the band gap of a semiconductor), there is no possibility of finding an e^- there !
- At 0K --> distribution is rectangular (all states above E_F are empty, below are filled)
- At $T > 0$ K, some probability exists for states above the Fermi level to be filled.
- Distribution is always symmetrical about E_F for all temperatures

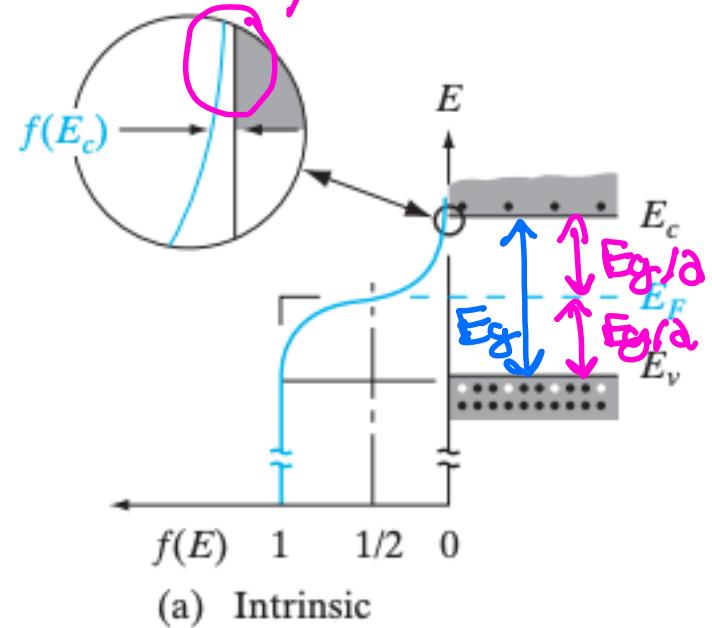
The Fermi–Dirac distribution function.



The Fermi Level in Intrinsic Materials

- There is only one Fermi level in a system at thermal equilibrium
- E_F depends on the # of electrons and holes present in a system
- For intrinsic material, we know # of h^+ in E_v = # of e^- in E_c
 - Fermi level in middle of bandgap
 - $E_c - E_F = E_c - E_i = E_g/2$
- Distribution function has values within E_g , but these states are unavailable
- There is some 'tail' probability of the e^- being found higher in E_c
- Equal to 'tail' probability of h^+ of $[1 - f(E)]$ extending into E_v

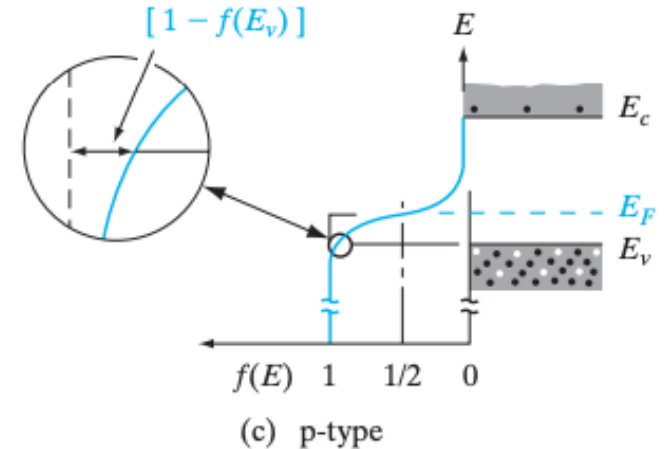
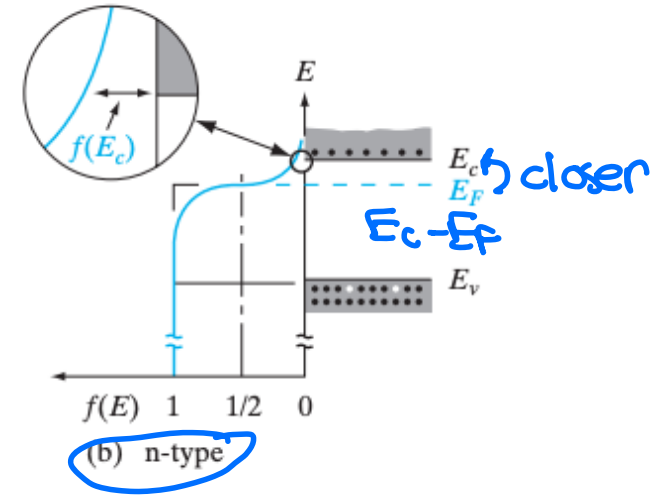
'Tail' probability e^- in higher energy state in E_c



Source: Textbook

The Fermi Level in Extrinsic Materials

- In doped materials, the # of 'free' holes and electrons is not equal
- n-type: more e^- in E_c than h^+ in E_v
 - Therefore, E_F is closer to E_c edge
 - The value of $(E_c - E_F)$ indicates how strongly n-type the material is
- p-type: more h^+ in E_v than e^- in E_c
 - Therefore, E_F is closer to E_v edge
 - The value of $(E_F - E_v)$ indicates how strongly p-type the material is



Source: Textbook

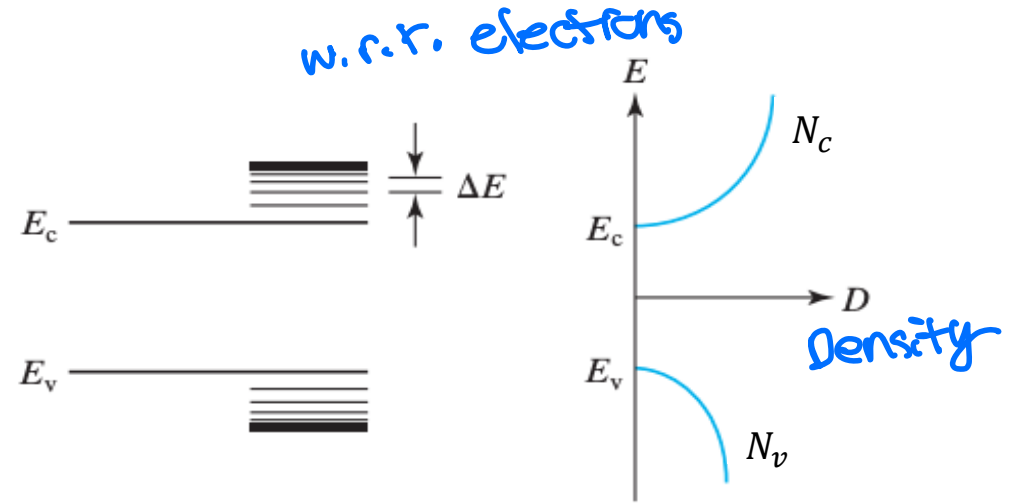
2) Density of States

- Imagine counting the # of states in a small range of energy in the conduction band, ΔE :

$$N(E) = \frac{\text{number of states in } \Delta E}{\Delta E \times \text{volume}}$$

N_c , effective density of states at the edge of E_c

N_v , effective density of states at the edge of E_v



- Why does D.O.S. increase as we move up in the bands?

As E increases, more quantum states available for e^- to occupy

Density of states in Si conduction & valence band

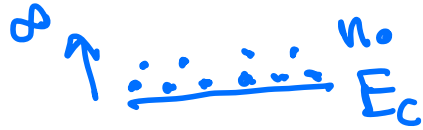
- Counting states (i.e. “stadium seats per row”) in App. IV:

$$N(E) = \frac{\sqrt{2}}{\pi^2} \left(\frac{m^*}{\hbar^2} \right)^{3/2} E^{1/2}$$

- Most important feature is $\sim E^{1/2}$ (more states at higher E)!
- What is E with respect to? Any states in the band gap?
w.r.t. E_c or E_v , no states available in the bandgap
- What is m^* here?
effective mass

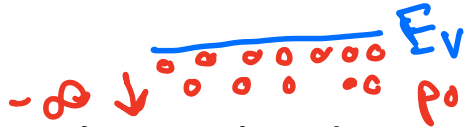
Carrier Concentrations

- So how do we calculate electron & hole concentrations knowing the density of states and probability of occupation?



$$n_0 = \int_{E_C}^{\infty} f(E) N(E) dE$$

in conduction band



- This is the density of electrons in the C-band. What about holes in the V-band?

$$p_0 = \int_{-\infty}^{E_V} [1 - f(E)] N_v(E) dE$$

- This is usually a tough numerical integral, but we can approximate below if E_F is well within the bandgap!

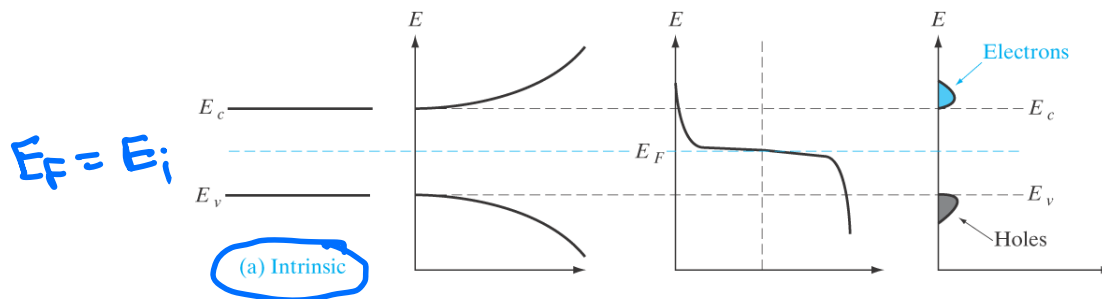
$$n_0 \approx N_c e^{-(E_C - E_F)/kT}$$

where

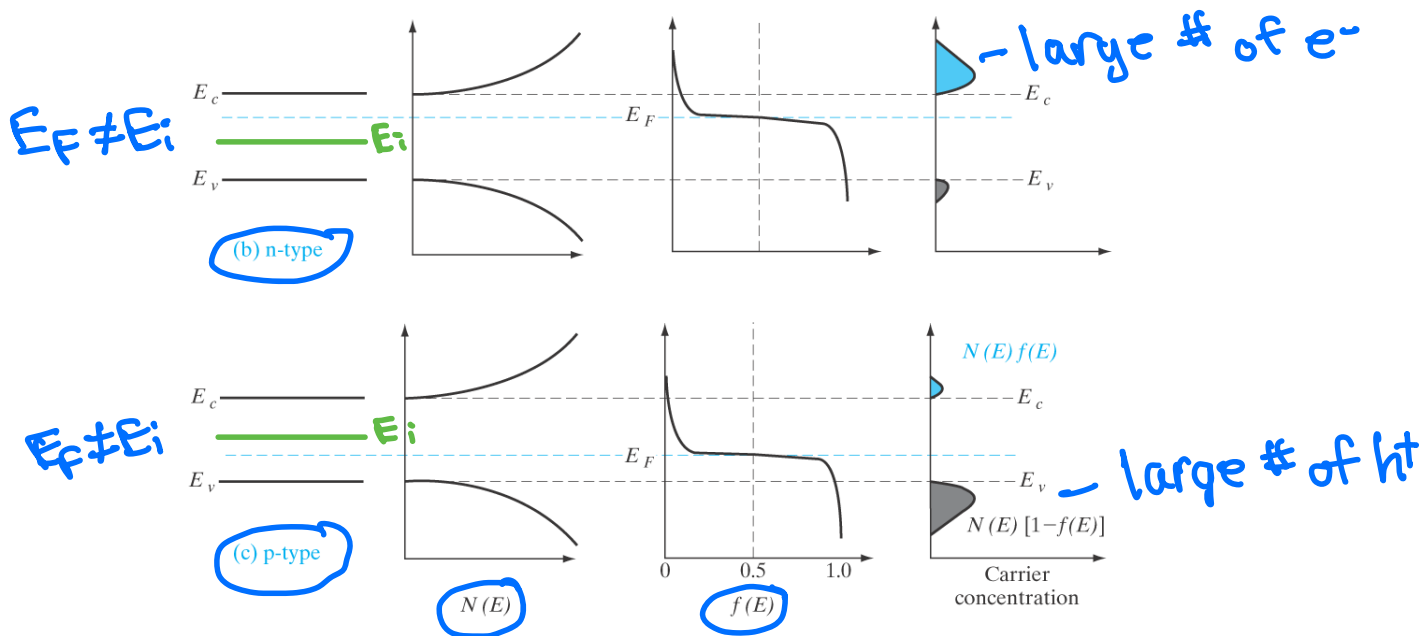
$$N_c = 2 \left(\frac{2\pi m_n^* kT}{h^2} \right)^{\frac{3}{2}}$$

Visualizing the $f(E)$, density, and carrier concentration

- Where is E_F in intrinsic material?



- What happens with E_F when we start doping the material?



Carrier Concentrations Continued

- We can write similarly for holes:

$$p_0 = N_v e^{-(E_F - E_v)/kT}$$

where

$$N_v = 2 \left(\frac{2\pi m_p^* kT}{h^2} \right)^{\frac{3}{2}}$$

- Recall, $n_i = p_i$, and we can write

$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$p_i = N_v e^{-(E_i - E_v)/kT}$$

Note: the product of n_0 and p_0 at equilibrium is constant for a particular material and temperature, even if doping is varied! (e.g. $n_0 \neq p_0$)

- Called the Mass-action Law

- Recall if the material is intrinsic, $E_F = E_i$ where

$$\rightarrow E_c - E_i = \frac{E_g}{2}$$

$$n_i^2 = n_i p_i = (N_c e^{-(E_c - E_i)/kT}) (N_v e^{-(E_i - E_v)/kT}) = N_c N_v e^{-\frac{E_g}{kT}}$$

$$n_i^2 = n_0 p_0 = (N_c e^{-(E_c - E_F)/kT}) (N_v e^{-(E_F - E_v)/kT}) = N_c N_v e^{-\frac{(E_c - E_v)}{kT}} = N_c N_v e^{-\frac{E_g}{kT}}$$

Given $n_i = p_i$,

$$n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}}$$

Source: Textbook

Carrier Concentrations Continued

- We can conveniently rewrite these equations,

$$n_0 = N_c e^{-(E_c - E_F)/kT} \longrightarrow \boxed{n_0 = \underline{n_i} e^{(E_F - E_i)/kT}} \quad (3-25a)$$

$$p_0 = N_v e^{-(E_F - E_v)/kT} \longrightarrow \boxed{p_0 = n_i e^{(E_i - E_F)/kT}} \quad (3-25b)$$

- Convenient because we can easily find n_i for a given material and temperature

Problem: Solving for carrier concentration

A Si sample is doped with 10^{16} Boron atoms per cm^3 . What are the electron and hole concentrations at room temperature? Assume lights off. Is this n- or p-type material? Where is the Fermi level, E_F ?

Boron is group III \rightarrow acceptor \rightarrow p-type material
 $N_A = 10^{16} \text{ cm}^{-3} \rightarrow$ so $p_0 \approx N_A = 10^{16} \text{ cm}^{-3}$ * $N_A \gg n_i(\text{Si}) \approx 1.5 \times 10^{10} \text{ cm}^{-3}$

Mass action law: $n_0 p_0 = n_i^2 \rightarrow n_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10} \text{ cm}^{-3})^2}{10^{16} \text{ cm}^{-3}}$

$n_0 = 2.25 \times 10^4 \text{ cm}^{-3}$

$p_0 = n_i e^{(E_i - E_F)/kT} \rightarrow E_i - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$

$E_i - E_F = (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) \ln\left(\frac{10^{16} \text{ cm}^{-3}}{1.5 \times 10^{10} \text{ cm}^{-3}}\right) \approx 0.36 \text{ eV}$

$\therefore E_i - E_F \approx 0.36 \text{ eV}$

